Physics III
ISI B.Math
Mid Semestral Exam : October 4, 2010

Total Marks: 60
Time: 2 hours 30 min
Answer any 4 questions.

1.(i) Suppose instead of the Coulomb force law, one found experimentally that the force between any two charges q_1 and q_2 was

$$\mathbf{F_{12}} = \frac{q_1 q_2}{4\pi\epsilon_0} \frac{(1 - \sqrt{\alpha r_{12}})}{{r_{12}}^2} \hat{\mathbf{r}}_{\mathbf{12}}$$

where α is a constant.
(a) Write down the appropriate electric field $\mathbf{E}(\mathbf{r})$ due to a charge q at the origin. (2)
(b)Choose a path around this point charge and compute the line integral $\oint \mathbf{E} \cdot \mathbf{dl}$. Compare with the Coulomb result.(4)
(c) Find $\oint \mathbf{E} \cdot \mathbf{da}$ over a spherical surface of radius r_1 with the point charge at the centre. Compare with the Coulomb result.(4)
(ii) Write down an expression for the volume charge density of an electric dipole, consisting of a point charge $-q$ at the origin and a point charge $+q$ at $\mathbf{a}$.(2)
(iii) Evaluate $\nabla(\frac{\mathbf{1}}{|\mathbf{r}-\mathbf{r}'|})$(3).

2.(a) A uniform line charge λ per unit length is placed on an infinite straight wire, a distance d above a grounded conducting plane (let us say the wire runs parallel to the x-axis and directly above it, and the conducting plane is the x-y plane).
(i) Find the potential in the region above the plane.(5)
(ii) Find the charge density σ induced on the conducting plane.(5)
(b) Find the energy of a uniformly charged spherical shell of total charge Q and radius R. (5)

3.a) Consider a hollow spherical conducting shell of radius R with a charge $-q$ uniformly distributed on it. If a charge $+q$ is placed inside the sphere at

the centre, i) will it be in equilibrium? ii) if it is in equilibrium, is the equilibrium stable? (iii) How will the answers to i) and ii) change if the charge $+q$ is placed at a distance a from the centre where $a < R$? (iv) How will the answers to i), ii) and iii) change if the charge on the shell is $+q$ instead of $-q$?. Give reasons for your conclusions. (3)
b) Which of the following statements are true in general in the context of electrostatics and magnetostatics ?
i)$\nabla \times \mathbf{D} = 0$, ii)$\nabla \times \mathbf{B} = 0$ iii)$\nabla \times \mathbf{E} = 0$ iv) $\nabla \times \mathbf{P} = 0$
Do any of your above conclusions get modified when we specialize to the case of homogeneous linear dielectrics? (2)
In a region of space where there is no free charge, which of the following statements are true?
i) $\nabla \cdot \mathbf{E} = 0$ ii) $\nabla \cdot \mathbf{D} = 0$ iii) $\nabla \cdot \mathbf{P} = 0$ iv) $\nabla \cdot \mathbf{B} = 0$ (2)
c) A metal sphere of radius a carries a charge Q It is surrounded out to radius b by linear dielectric material of permittivity ϵ. Find the potential at the centre (relative to infinity) and the bound charge density. (8)

4. Two infinite grounded metal plates lie parallel to the xz plane, one at $y = 0$ and the other at $y = a$. The left end at $x = 0$ is closed off with an infinite strip insulated from the two plates and held at a constant potential V_0.
a) Use the method of separation of variables to show that the potential inside the slot can be written in the form

$$V(x, y) = \sum_{n=1}^{\infty} C_n e^{-\frac{n\pi x}{a}} \sin(\frac{n\pi y}{a})$$

where C_n are constants. (6)
(b) Use the boundary conditions to determine C_n.(5)
(c) Determine the charge density $\sigma(y)$ on the strip at $x = 0$, assuming it is a conductor at potential V_0(4).

5. (a) Show that kinetic energy of a charged particle is conserved when it is moving in a pure magnetic field.(3)
(b)A steady current I flows down a long cylindrical wire of radius a. Find the magnetic field, both inside and outside the wire if (i) the current is uniformly distributed over the outside surface of the wire.(3)
(ii) the current is distributed in such a way that the current density $\mathbf{J}$ is

proportional to s, the distance from the axis.(3)
(c) Rewrite the continuity equation

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$$

in integral form by integrating over a volume $\mathcal{V}$ and show that it implies the conservation of charge. (3)
(d) Consider a uniform magnetic field $\mathbf{B}_0 = B_0\mathbf{z}$. Which of the following vectors qualify as a vector potential corresponding to this field? i) $\mathbf{A} = (0, xB_0, 0)$, ii)$\mathbf{A} = (0, yB_0, 0)$, iii) $\mathbf{A} = (-\frac{1}{2}yB_0, \frac{1}{2}xB_0, 0)$. Is the vector potential unique? If not, to what extent is it arbitrary ?(3)